23 COSMOLOGY

Student Learning Outcomes (SLOs)

The student will

- Explain the term luminosity [as the total power of radiation emitted by a star].
- Apply the inverse square law for radiant flux intensity [F in terms of the luminosity L of the source $F = \frac{L}{4\pi d^2}$].
- Define and apply standard candles [Explain the use of standard candles to determine distances to galaxies].
- Explain blackbody radiation and apply Wien's displacement law to solve problems [$\lambda_{max}T$ = constant to estimate the peak surface temperature of a star]
- Apply the Stefan-Boltzmann law to solve problems [$L = 4\pi r^2 \times \sigma T^4$ to solve problems]
- estimate the radius of a star [applying Wien's displacement law and the Stefan-Boltzmann law]
- Explain that the lines in the emission and absorption spectra from distant objects show an increase in wavelength from their known values.
- explain why redshift leads to the idea that the Universe is expanding [include using $\frac{\Delta\lambda}{\lambda} \approx \frac{\Delta f}{f} \approx \frac{v}{c}$ for the redshift of electromagnetic radiation from a source moving relative to an observer to solve problems relating to the expanding universe].
- State and explain Hubble's law and how it leads to the Big Bang theory.

Electromagnetic radiations are playing a vital role in facilitating us during research. It can give us in depth information of a living or non-living body by penetrating into it. Electromagnetic radiations also give us information of far-off outer space or universe.

Electromagnetic energy travels in the form of waves and spans a broad spectrum ranging from very long radio waves to very short gamma rays. The human eye can only detect a small portion of this spectrum, called visible light. NASA's scientific instruments use the full range of the electromagnetic spectrum to study the Earth, the solar system, and the universe beyond.

Cosmology is the study of the origin, development, structure, history, and future of the entire universe. Radiation plays a significant role in cosmology, particularly in understanding residual radiation from the Big Bang. These aspects help us study the universe's origins, evolution, and structure.

23.1 BLACK BODY RADIATION

A black-body is an ideal object which is perfectly opaque and non-reflecting.

- It absorbs all the radiation that falls on it.
- It is also a good emitter.

A black-body is a theoretical object; however, stars are the best examples. The radiation emitted from a black-body has a characteristic spectrum that is determined by the temperature.

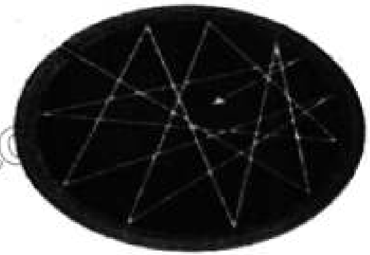


Figure 23.1: An ideal blackbody.

Wien's Displacement Law

This law helps understand thermal radiation, temperature and emission spectra. It shows that hotter objects emit shorter wavelengths (e.g., blue light), while cooler objects emit longer wavelengths (e.g., red light).

Wien's displacement law relates the observed wavelength of light emitted by a star to its surface temperature (T). It states that:

The black body radiation curve for different temperatures peaks at a wavelength, which is inversely proportional to the temperature.

This relation can be written as:

$$\lambda_{max} \propto \frac{1}{T}$$

or

$$\lambda_{max} = \text{constant} \times \frac{1}{T}$$

λ_{max} is the maximum wavelength emitted by the star at the peak intensity, as shown in Fig. 23.2. Here the constant is equal to 2.9×10^{-3} m K. Thus, the equation for Wien's displacement law is given by:

$$\lambda_{max} T = 2.9 \times 10^{-3} \text{ m K} \qquad (23.1)$$

This equation shows that:

- The higher the temperature of a body, the shorter the wavelength at the peak intensity. Therefore, hotter stars tend to appear white or blue and cooler stars tend to appear red or yellow.
- The higher the temperature of a body, the greater the intensity of the radiation at each wavelength.

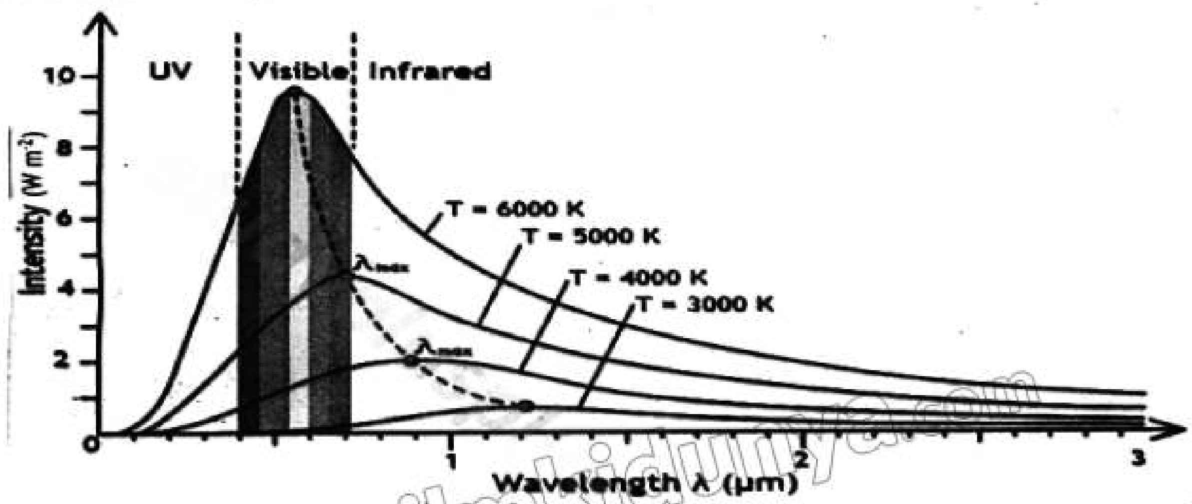


Figure 23.2: The intensity-wavelength graph shows the relation of temperature to the peak wavelength for four different stars.

23.2 LUMINOSITY AND RADIANT FLUX INTENSITY

Luminosity and radiant flux intensity are two different quantities related to electromagnetic radiation. Let us understand each term separately, before exploring the relationship between them.

Luminosity

In astronomy, luminosity is the total amount of electromagnetic energy emitted per unit time by a star or other celestial objects. This represents the power output of radiation emitted by a star.

Luminosity is a measure of the total power output of radiation emitted by a star.

In SI units, luminosity is measured in joule per second ($J\ s^{-1}$), or watt (W).

Luminosity depends on temperature and surface area of the star. The relationship between these two quantities is known as the Stefan-Boltzmann Law, which states that:

The total energy emitted by a blackbody per unit area per second is proportional to the fourth power of the absolute temperature of the body.

When considering a star to be a completely black body, the Stefan-Boltzmann law can be applied to find the value for luminosity (L) for a black body. i.e.,

$$L = A\sigma T^4 \qquad \text{------- (23.2 a)}$$

Where A is the surface area, T is the temperature (in kelvins) and σ is the Stefan-Boltzmann constant, with a value of $5.67 \times 10^{-8}\ Wm^{-2}\ K^{-4}$. The surface area of a star (being sphere) is equal to $4\pi r^2$, so Eq. (23.2 a) can also be written as:

$$L = 4\pi r^2 \sigma T^4 \qquad \text{------- (23.2 b)}$$

If two stars have same temperature, then the star with double radius will have approximately four times the luminosity.

Radiant Flux Intensity

Light sources (such as stars) that are further away from the Earth appear fainter because the light they emit is spread out over a larger area. The moment the light leaves the surface of the star, it begins to spread out uniformly through a spherical shell (with an area of $4\pi r^2$). The radius *r* of this sphere is equal to the distance *d* between the star and the Earth. By the time the radiation reaches the Earth, it has been spread over an area of $4\pi d^2$.

The luminosity per unit area measured on the surface of the Earth is known as radiant flux intensity.

Thus, the relationship between the luminosity (L) and radiant flux intensity (F) can be expressed as:

$$F = L/A \qquad \text{------- (23.3 a)}$$

$$F = \frac{L}{4\pi d^2} \qquad \text{------- (23.3 b)}$$

where A is the area of the illuminated surface. In SI units, radiant flux intensity is measured in $W\ m^{-2}$. Eq. (23.3) shows that the radiant flux follows an **inverse square law**: when light is twice as far away, it has spread over four times the area, resulting in a four-fold decrease in intensity.

Consider a point source of light of luminosity L that radiates equally in all directions, as shown in Fig. 23.3. A hollow sphere centered on the point would have its entire interior surface illuminated. As the radius increases, the surface area also increases, and the constant luminosity has more surface area to illuminate, leading to a decrease in observed brightness (radiant flux intensity).

For Your Information

Value of solar luminosity is: 3.84×10^{26} W

A greater radiant flux intensity (larger F) indicates that the star is closer to the Earth (smaller d).

The luminosity is the total power output of the star, whereas the radiant flux intensity is what is measured on Earth.

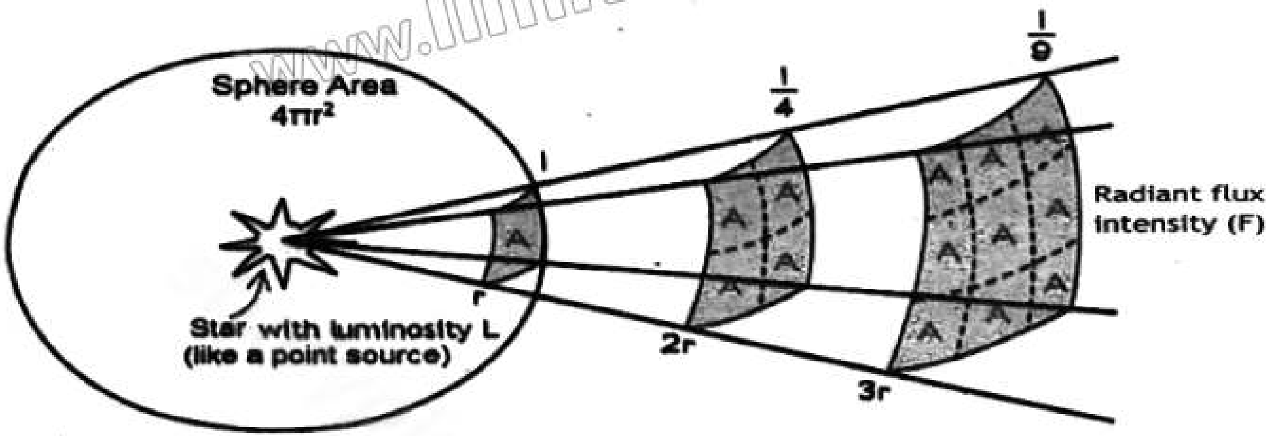


Figure 23.3: Inverse Square Law of radiant flux intensity.

Example 23.1: A star has a known luminosity of 9.7×10^{27} W. Observations of the star show that the radiant flux intensity of light received on Earth from the star is 114 nW m^{-2}. Determine the distance of the star from Earth.

Given: Luminosity = L = 9.7×10^{27} W

Radiant flux intensity = F = 114 nW m^{-2} = 114×10^{-9} W m^{-2}

To Find: Distance from the Earth = d = ?

Solution: We use the equation, $$F = \frac{L}{4\pi d^2}$$

By rearrange for distance d, we get: $$d = \sqrt{\frac{L}{4\pi F}}$$

By putting values, we get: $$d = 8.2 \times 10^{16}\ m$$

Assignment 23.1

What is the relative luminosity of the objects A and B if: T_A = 100 K, T_B = 200 K, R_A = 10 m and R_B = 5 m.

23.3 STANDARD CANDLES: A Distance Indicator

Measuring astronomical distances accurately is an extremely difficult task. A direct distance measurement is only possible if the object is close enough to the Earth. For more distant objects, indirect methods must be used. Standard candles are useful for this purpose.

In astronomy, a standard candle is a source with a known luminosity. A standard candle is defined as:

An astronomical object which has a known luminosity due to a characteristic quality possessed by that class of object.

If the luminosity of a source (i.e., standard candle) is known, then the distance can be calculated by using the Eq. (23.3 b), which is based on how bright (i.e., the measured radiant flux intensity) it appears from Earth.

Examples of standard candles are: Cepheid variable stars and Type 1A supernovae. Each standard candle method can measure distances within a certain range. Organizing the data and measurements from each method allows astronomers to build up a larger picture of the universe, from nearby stars to distant galaxies. This is known as the cosmic distance ladder.

For Your Information

Edwin Hubble used Cepheid to determine the distances of "nebulae" and derive the Hubble law. Nebulae are interstellar clouds of gas and dust.

Estimating the Radius of Stars

The radius of a star can be estimated by combining Wien's displacement law and the Stefan-Boltzmann law. The procedure for this is as follows:

- Find the surface temperature of the star by using Wien's displacement law.
- Find the luminosity of the star by using the inverse square law of flux (if the radiant flux and stellar distance are given).
- Then, using the Stefan-Boltzmann law, the stellar radius can be obtained.

Example 23.2: The spectrum of the star Rigel in the constellation of Orion peaks at a wavelength of 263 nm, while the spectrum of the star Betelgeuse peaks at a wavelength of 828 nm. Find the surface temperature of these two stars. Which of these two stars is cooler?

Given: For the star Rigel: $\lambda_{max} = 263 \text{ nm} = 263 \times 10^{-9} \text{ m}$

For the star Betelgeuse: $\lambda_{max} = 828 \text{ nm} = 828 \times 10^{-9} \text{ m}$

To Find: Surface temperature for the star Rigel: $T = ?$

Surface temperature for the star Betelgeuse: $T = ?$

Which of the two stars is cooler = ?

Solution: We use the equation,

$$\lambda_{max} T = 2.9 \times 10^{-3} \text{ m K}$$

By rearrange for distance d, we get:

$$T = 2.9 \times 10^{-3} / \lambda_{max}$$

For the star Rigel: $T = 2.9 \times 10^{-3} / \lambda_{max} = 2.9 \times 10^{-3} / 263 \times 10^{-9} = 11026 \text{ K}$

For the star Betelgeuse: $T = 2.9 \times 10^{-3} / \lambda_{max} = 2.9 \times 10^{-3} / 828 \times 10^{-9} = 3502 \text{ K}$

As, Betelgeuse has surface temperature of 3500 K, therefore, it is cooler than Rigel.

Assignment 23.2

Betelgeuse is our nearest red giant star. It has a luminosity of 4.49×10^{31} W and emits radiation with a peak wavelength of 850 nm. Calculate the ratio of the radius of Betelgeuse r_B to the radius of the Sun r_S.

23.4 SPECTRA OF LIGHT

Astronomers are very limited in how they can investigate objects in the space. All of the techniques used by astronomers involve analysing the light emitted from the star, or galaxy. One of these techniques involves analysing the emission and absorption spectra of stars.

Elements in the star (mostly hydrogen and helium) absorb some of the emitted wavelengths. Therefore, characteristic lines are present when the spectrum is analysed. Every element has a unique set of absorption and emission lines. The pattern of lines is known as a spectral signature. The absorption and emission spectra of each element are inverses of each other: The wavelengths of a particular element's absorption lines are the same as the wavelengths of its emission lines. Astronomers can compare the spectrum of a celestial object or material with the spectra of known elements and molecules to figure out what the object or material is made of.

Absorption Spectra: When light passes through a gas, atoms and molecules in the gas absorb certain colors, or wavelengths, of that light. The result is an absorption spectrum: a rainbow with dark absorption lines.

Emission Spectra: The same gas can glow, giving off very specific colors to form an emission spectrum with bright lines known as emission lines.

Spectra of Light from Stars and Galaxies

When astronomers observe light from distant galaxies, they observe differences in the spectral lines to the light from the Sun. The lines in the emission and absorption spectra from distant objects show an increase in wavelength from their known values. These lines appear to be shifted sightly towards the red end of the spectrum but the lines have the same characteristic pattern, meaning the element can still be easily identified.

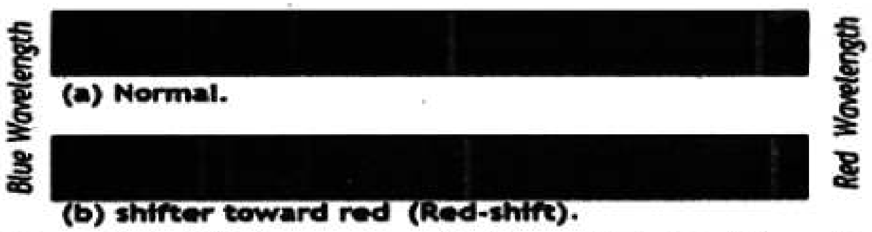


Figure 23.4: Emission spectra of hydrogen (a) spectral lines of hydrogen measured from a source in laboratory. (b) spectral lines of hydrogen measured from a distant galaxy.

Red-shift and Speed

Astronomers see red-shift in all galaxies. It is due to the reason that 'the space between the Earth and the galaxies is expanding'. This expansion has the effect of increasing the wavelength of the light from these galaxies, shifting them towards the red end of the spectrum. The more red-shifted the light from a galaxy is, the faster the galaxy is moving away from Earth. Recall that the Doppler effect is defined as:

There is apparent change in wavelength or frequency of the radiation from a source due to its relative motion toward or away from the observer.

Doppler effect of light can observe when spectra of distant stars and galaxies are observed, this is known as:

- Redshift, if the object is moving away from the Earth.
- Blueshift, if the object is moving towards the Earth.

Redshift is defined as:

The fractional increase in wavelength (or decrease in frequency) due to the source and observer receding from each other.

Red-shift can be calculated using:

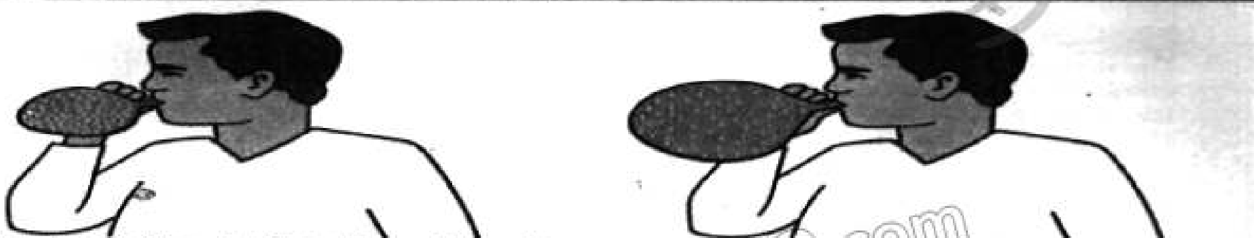
$$\frac{\Delta\lambda}{\lambda} = \frac{\Delta f}{f} = \frac{v}{c} \quad \text{(23.4)}$$

Where, $\Delta\lambda$ is shift in wavelength, λ is wavelength emitted from the source, Δf is shift in frequency, f is frequency emitted from the source, v is speed of recession and c is speed of light in a vacuum.

Expanding Universe

After the discovery of Doppler redshift, astronomers began to realise that almost all the galaxies in the universe are receding. This led to the idea that the space between the Earth and the galaxies must be expanding. This expansion stretches out the light waves as they travel through space, shifting them towards the red end of the spectrum. The more red-shifted the light from a galaxy is, the faster the galaxy is moving away from Earth.

Activity 23.1: A balloon inflating is similar to the stretching of the space between galaxies



When a dotted balloon is inflated then all the dots move away from each other as the rubber stretches. The expansion of the universe can be compared to dots on an inflating balloon. Just like the dots, the galaxies move away from each other.

23.5 HUBBLE'S LAW AND THE BIG BANG THEORY

Edwin Hubble investigated the light spectra emitted from a large number of galaxies. He used redshift data to determine the galaxy's recessional velocity (v), and standard candles to determine the distances (d) between the galaxy and Earth. From these measurements, he formulated a relationship, now known as Hubble's Law. Hubble's Law states:

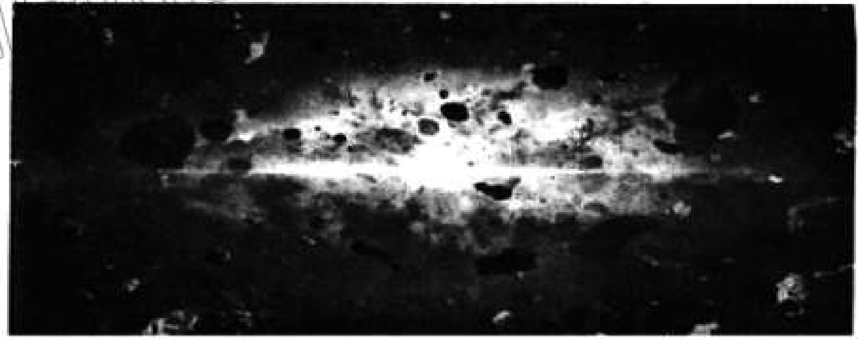


Figure 23.5: The Big Bang Theory explains how the universe began 13.8 billion years ago.

The recession speed of a galaxy is directly proportional to its distance from the Earth.

Expression for the Hubble's Law can be written as:

$$v = H_0 d \qquad (23.5)$$

Where, H_0 is Hubble's constant, or the rate of expansion of the universe (s^{-1}). The Eq. (23.5) shows that:

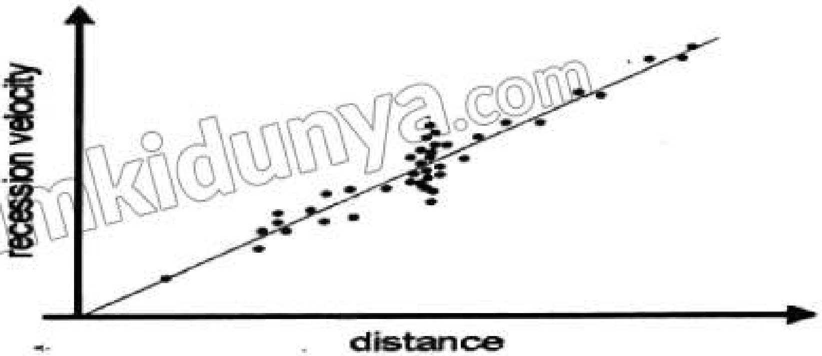
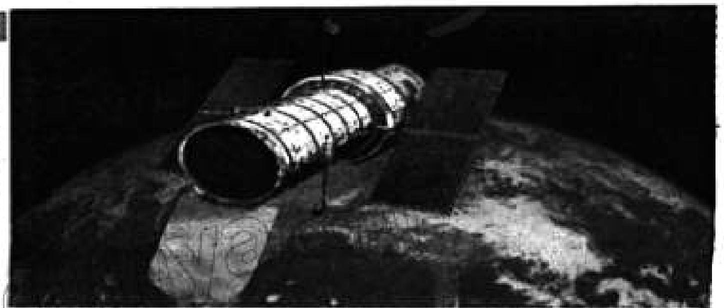


Figure 23.6: Graph of recession velocity against distance.

- The further away a galaxy, the faster it's recession velocity (v).
- The gradient of a graph of recession velocity against distance is equal to the Hubble constant (i.e., $v/d = H_0$), as shown in Fig. 23.6.

Hubble Space Telescope

The Hubble Space Telescope is a space-based observatory launched in 1990. It orbits very low in earth orbit, approximately 340 miles (540 km) above Earth's surface. It observes ultraviolet, visible, and near-infrared light. It contributed significantly to understanding galaxy evolution, star formation, and cosmology. It has also determining the rate of expansion of the universe.



Age of the Universe

If the galaxies are moving away from each other, then they must've started from the same point at some time in the past, as shown in Fig. 23.7. If this is true, the universe likely began in an extremely hot, dense singular point which subsequently began to expand very quickly. This idea is known as the Big Bang theory. According to the Big Bang theory:

About 13.8 billion years ago the whole universe was a very small, extremely hot and dense region. From this tiny point, the whole universe expanded outwards to what exists today.

Redshift of galaxies and the expansion of the universe are now some of the most prominent pieces of evidence to suggest this theory is true. The data from Hubble's law can be extrapolated back to the point where the universe started expanding, i.e., the beginning of the universe, as shown in Fig. 23.7. Therefore, the age of the universe T_o is equal to:

$$T_o = \frac{1}{H_o} \quad (23.6)$$

Figure 23.7: Tracing the expansion of the universe back to the beginning of time leads to the idea the universe began with a Big-Bang.

Presently estimated age of the universe ranges from 13 to 14 billion years. There is still some discussion about the exact age of the universe, therefore, finding accurate measurements for the Hubble's constant is a top priority for cosmologists.

James Web Space Telescope

It is launched in 2021. It orbits approximately 1.5 million kilometers from Earth.

It observes near-infrared and mid-infrared light, studying galaxy formation, star birth, and planetary systems. Its main objectives are to studying the universe's early stages, to understanding galaxy formation and evolution, observing star formation and planetary system development.

Example 23.3: A specific line in the spectrum of light obtained from a source in laboratory has a frequency of 4.570×10^{14} Hz. The same line in the spectrum of light from a distant galaxy has a frequency of 4.547×10^{14} Hz. What speed is the distance galaxy moving in relation to the Earth? Is it moving towards or away from the Earth?

Given: frequency of lab source: $f = 4.570 \times 10^{14}$ Hz
frequency of distant galaxy: $f' = 4.547 \times 10^{14}$ Hz

To Find: Speed of the galaxy: v = ?

Solution: Shift in frequency: $\Delta f = (4.570 - 4.547) \times 10^{14} = 2.3 \times 10^{12}$ Hz

Now we use the equation: $\frac{\Delta f}{f} = \frac{v}{c}$

Rearrange for speed v and putting values, we get:

$$v = \frac{c\Delta f}{f} = \frac{(3\times10^{8})(2.3\times10^{12})}{4.57\times10^{14}} = 1.5\times10^{6}\ m\ s^{-1}$$

The observed frequency is less than the emitted frequency (the light from a laboratory source), therefore, the source is receding from the Earth at $1.5 \times 10^{6}\ m\ s^{-1}$.

Assignment 23.3

A galaxy is found to be moving away with a speed of $2.1 \times 10^{7}\ m\ s^{-1}$. The galaxy is at a distance of 9.5×10^{24} m from Earth. Assuming the speed has remained constant, what is the age of the universe in years?

SUMMARY

- **Luminosity** is a measure of the total power output of radiation emitted by a star. In SI units, luminosity is measured in joule per second or watt.
- **Stefan-Boltzmann Law:** the total energy emitted by a black body per unit area per second is proportional to the fourth power of the absolute temperature of the body.
- **Blackbody** is an idealized object that is perfectly opaque and non-reflecting.
- Luminosity per unit area measured on the surface of the Earth is known as **radiant flux intensity.**
- **A standard candle** is defined as: an astronomical object that has a known luminosity due to a characteristic quality possessed by that class of object.
- **Wien's displacement law** states that: The black body radiation curve for different temperature peaks at a wavelength, which is inversely proportional to the temperature.
- **Redshift** is defined as: the fractional increase in wavelength (or decrease in frequency) due to the source and observer receding from each other.
- **Hubble's Law** states: the recession speed of galaxies moving away from Earth is proportional to their distance from the Earth.
- According to the **Big Bang theory:** about 13.8 billion years ago the whole Universe was a very small, extremely hot and dense region. From this tiny point, the whole Universe expanded outwards to what exists today.
- **Estimated age of the universe** ranges from 13 to 14 billion years.

Formula Sheet

$\lambda_{max} T = 2.9 \times 10^{-3}$ m K	$L = A\sigma T^{4}$	$F = L/A$
$\frac{\Delta\lambda}{\lambda} = \frac{\Delta f}{f} = \frac{v}{c}$	$v = H_0 d$	$T_o = \frac{1}{H_o}$

EXERCISE

Multiple Choice Questions

Encircle the Correct option.

1) SI unit of Luminosity is
   A. watts B. joules C. newtons/meter D. cd/m
2) Luminosity is associated with energy produced by
   A. objects B. celestial bodies C. light source D. fluorescent lamp
3) Luminosity depends on
   A. Size of the star B. Temperature of the star
   C. Colour of the star D. Option 1 and 2
4) Which law states that brightness of a source is inversely proportional to the square of its distance
   A. Law of brightness B. Direct Square Law of brightness
   C. Inverse Square Law of brightness D. Inverse Square Law of light
5) SI unit of Hubble's constant (H_o) is
   A. s^{-1} B. m^{-1} C. m D. $m\ s^{-1}$
6) Which statement about the Big-Bang theory is not correct:
   A. There was a giant explosion known as the big bang.
   B. This caused the universe to expand from a single point.
   C. Each point in the universe expands away from the others.
   D. The further away galaxies are the slower they are moving.

Short Questions

1) What is meant by blackbody radiation?
2) What is the relationship between luminosity and radiant flux intensity?
3) What is the inverse square law for radiant flux intensity?
4) What is meant by standard candles? Give any two examples.
5) What is Wien's displacement law?
6) Define Stefan-Boltzmann law.
7) How can we estimate the radius of a star by applying Wien's displacement law and the Stefan-Boltzmann law?
8) How redshift leads to the idea that the Universe is expanding?
9) How does the expansion of the universe support the Big Bang theory?

10) How do different types of spectra (emission, absorption, continuous) provide insights into the composition and properties of celestial objects?

Comprehensive Questions

1) Explain the term luminosity and radiant flux intensity.
2) Discuss the use of standard candles to determine distances to galaxies.
3) Explain that how the radius of a star can be estimated by applying Wien's displacement law and the Stefan-Boltzmann law.
4) Why redshift leads to the idea that the Universe is expanding? Discuss.
5) State and explain Hubble's law and how it leads to the Big Bang theory.

Numerical Problems

1) There are two stars, let named A and B. Which star has the greater temperature? Given that: $L_A = L_B = 10^4$ J/s; $R_A = 10^4$ m; $R_B = 10^5$ m. (Ans: star A)

2) The Sun has a surface temperature of 6000K produces peak radiation of 420 nm. Find out the temperature of the Sirius if peak radiation of Sirius is 72 nm. (Ans: 35000 K)

3) If a source has a luminosity of 3.826×10^{26} W and is at a distance of 3 lightyear, what is the radiant flux intensity? (Ans: 3.78×10^{-6} W m^{-2})